Monte Carlo Simulation of Radiological Imaging Systems and the Recovery of the Poisson Distribution

Objective: To analyze Monte Carlo (MC) simulated data with regard to a) the type of distribution generated, b) the problem of Poisson distribution recovery, c) quantitative MC and d) a stopping criteria for MC simulations.

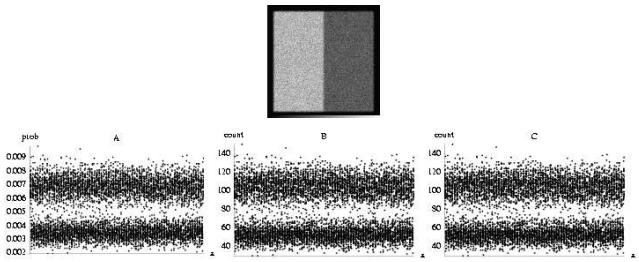
Summery: In order to perform this investigation, a MC simulation program which includes photon-specific forced detection/interaction VR techniques is used. By computing generalized linear model estimates of simulated distributions, we found that there exists a scaling factor which scales any uni-variate un-attenuated distribution into the corresponding Poisson distribution. If attenuation is present, we extend the simulated exponential mixture by an un-attenuated population and use the moments of this reference sample to calculate the scaling factor which recovers the complete finite Poisson mixture. The presented results could increase the potential applicability of MC simulations in nuclear medicine by performing quantitative MC simulation and reducing computational load by a count-based stopping criteria. As a further result of this investigation, we confirmed that the error introduced by the included VR techniques is marginal.

Methods: In MC, the characteristics of an imaging system are described by probability density functions (PDF's), and simulating is the act of sampling from those PDF's. Due to the very high number of samples necessary to produce statistically acceptable results, MC is almost impracticable for fully three-dimensional imaging systems without the use of variance reduction (VR) techniques. Due to the low geometric efficiency defined by collimator and aperture, VR by forced detection/interaction of the photon (besides other methods) is highly effective. VR is, however, biased and involves modifying the PDF's to increase the detection efficiency. To account for changes of the PDF's, a weighted probability which is associated with the photon is accumulated after detection rather than the photon event itself. Consequently, MC simulated data are small float numbers (mostly $\ll 1$) and do not (immediately) form a Poisson mixture. To study the nature of the distributions, a series of SPECT MC simulations has been performed.

By computing GLM estimates of the simulated distributions, we found that there exists a scaling factor

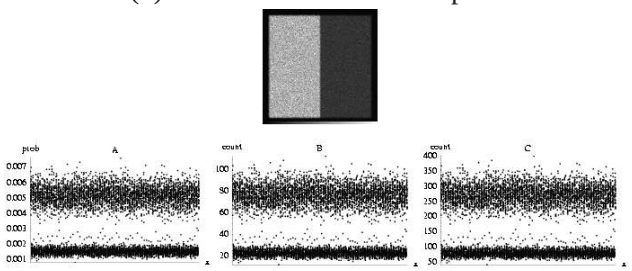
$$k = m / n^2 \qquad (1)$$

which scales any simulated uni-variate un-attenuated distribution into the corresponding Poisson distribution.



Simulated projection of two flat sources with disintegration ratio of 1:0.5 and corresponding frequency plot (A). With no attenuation present, scaling the complete data either by k_1 and k_2 leads to identical Poisson mixtures (B, C) of the complete data.

How does (1) behave if attenuation is present?

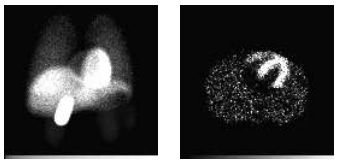


Simulated projection of two flat sources with equal relative disintegration but different attenuating media between source and camera and corresponding frequency plot (A). Scaling the complete data with k_1 and k_2 leads to different mixtures (B, C) whereby only k_1 (B) recovers the true Poisson mixture.

The results of these introductory experiments make the following remarkable modifications possible in the way MC is employed and could increase the potential applicability of MC simulations in nuclear medicine:

a) performing quantitative MC simulation and
b) introducing a count-based stopping criteria.

The figure below shows a with maximum likelihood expectation maximization (MLEM) reconstructed image of a MC simulated quantitative Tc-99m MIBI myocardial perfusion SPECT study. In addition to the accumulated attenuated photon probabilities, the un-attenuated photon probabilities from photons emitted inside the right lung tissue have been accumulated as well. This population is used to calculate k_* by which the exponential mixture has been scaled.



Quantitative Monte Carlo simulation and reconstructed image (attenuation corrected)

Proc. IEEE NSS/MIC, 2000.