Image Reconstruction

# Synergetically Generalized Expectation Maximization Reconstruction Algorithm for ECT

**Introduction:** The MLEM algorithm for reconstruction of nuclear medicine images has been extensively studied for its ability to produce superior reconstructions especially in low-count cases. There are numerous extensions to improve lesion contrast and smoothness based on Bayes or mixture models and to accelerate the algorithm to make it more practical. In all of these, the ML optimization criterion is constrained by some expectations regarding the spatial distribution of isotope concentration in the organ of interest.

**Objective:**

to transform the EM algorithm into a Langevin-type equivalent which describes appropriately the temporal behavior of quantum mechanical point-like particles in the field of quantum physics.

- a reconstruction algorithm for simultaneous segmentation, pattern recognition, and associative memory.
- image reconstruction in the higher context of non-equilibrium phase transitions.

## Transforming the EM Formalism into a Langevin-Type Equation

EM (estimating $\lambda$ from $g$) in additive form:

$$\lambda_j^{(t+1)} = \frac{\Sigma_i z_{ij}}{\Sigma_i c_{ij}} = \lambda_j^{(t)} \frac{\Sigma_i c_{ij} g_i / \langle c_i, \lambda^{(t)} \rangle}{\|c_j\|} = \lambda_j^{(t)} + \dot{\lambda}_j \quad \text{where} \quad \dot{\lambda}_j = \lambda_j^{(t)} \left( \frac{\Sigma_i c_{ij} g_i / \langle c_i, \lambda^{(t)} \rangle}{\|c_j\|} - 1 \right)$$

EM in Langevin-Type form (generalized):

$$\dot{\lambda} = \alpha^{(t)} (\mathrm{grad}_{\lambda^{(t)}} V(\lambda^{(t)}) + F_s^{(t)})$$

**Relaxation parameter**
acts as an acceleration factor and ensures that image values are non-negative

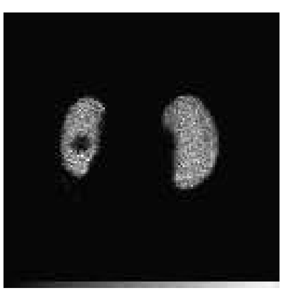
**Energy potential**

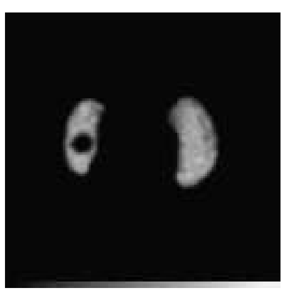
$$V(\lambda^{(t)}) = V_p(g|\lambda^{(t)}) + V_p(g^{(t)}) + V_p(\lambda^{(t)}|p(\Theta^{(t)})) + VF_d^{(t)} + \ldots$$

- $V_p(g|\lambda^{(t)})$: probability of obtaining the measurement vector $g$ — **EM criterion**
- $V_p(g^{(t)})$: prior probability of believing the image vector $\lambda$ — **Gibbs priors**
- $V_p(\lambda^{(t)}|p(\Theta^{(t)}))$: posterior probability of $g$ — **Data model**
- $VF_d^{(t)}$: additional forces (non-linear filters)

**Stochastic term**
describes fluctuating forces, usually Gaussian distributed

**Results:** The following two reconstructed images of the same high-count acquisition projection data set (anthropomorphic torso phantom, Data Spectrum Corp.) vary significantly in lesion contrast and smoothness as a result of different potential parameters.





Left: $V(\lambda^{(t)}) = V_p(g|\lambda^{(t)})$ (MLEM)

Right: $V(\lambda^{(t)}) = V_p(g|\lambda^{(t)}) + .7V_p(g^{(t)}) + .2V_p(\lambda^{(t)}|p(\Theta^{(t)}))$

**Discussion:** In the current state of investigation, initial applications of the proposed synergetically generalized EM algorithm for ECT show some encouraging results. Within this framework, classical (deterministic) approaches such as MLEM and GEM optimization can be thought as special cases. Introducing stochastic fluctuations into the system dynamics often leads to energetically better solutions although the system tends to oscillate between singular extrema if the force is not relaxed appropriate. Under pre-defined conditions of the external control parameters, the reconstructed image can be seen as a result of parametrized competition of the incorporated probability models and forces. From this point in view, the image under construction acts as a synergetic multi-particle system which is continuous in space and time. Handling the enormous capability of this approach, however, can be a problem.

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