Dose Minimization in Material-Selective Clinical CT with Photon-Counting Detectors

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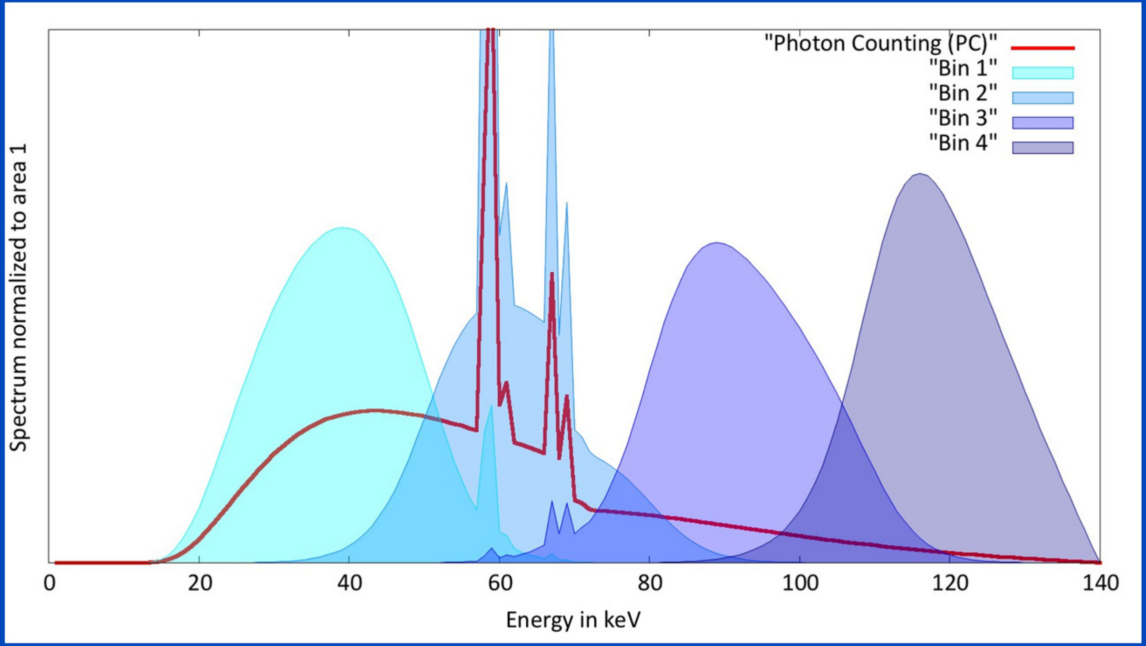
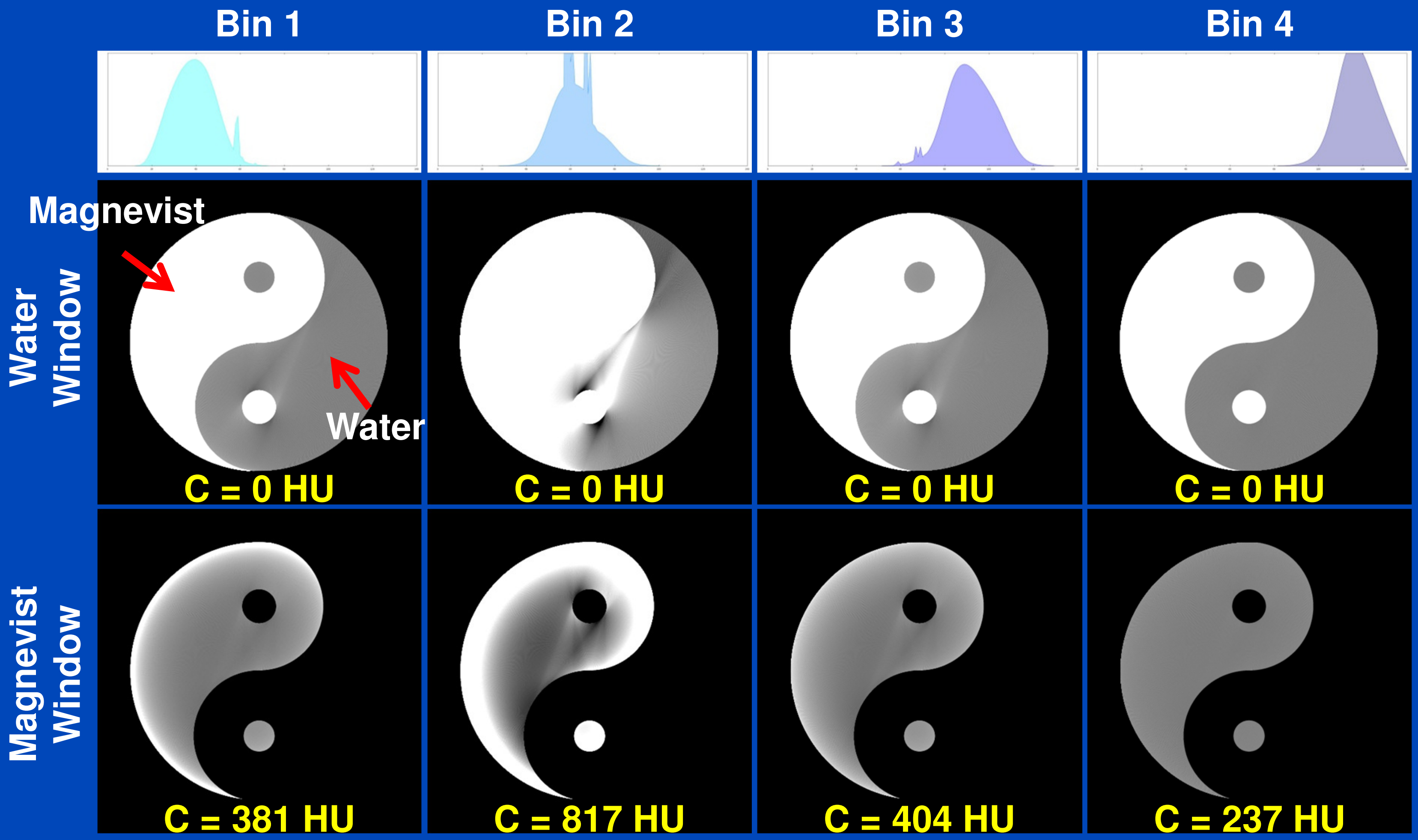
Aim

To optimize dose usage in energy-selective CT.



Red: Simulated 140 kV spectrum, prefiltered by 2 mm Aluminum and detected by a CdZnTe detector (1.4 mm thickness). Bin 1–4: Normalized spectrum of four (exemplarily) detected energy bins assuming a simple Gaussian spectra blurring with a standard deviation of 15 kV and centered at 35, 65, 95, and 125 keV.

Energy-Selective CT Images



W = 100 HU

(Images are water–precorrected)

Multiple Energy CT

- **Assumption: The object consists of M independent materials:**

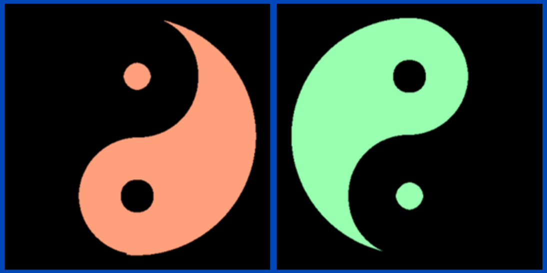
$$f_1(\boldsymbol{r})\psi_1(E) + f_2(\boldsymbol{r})\psi_2(E) + \ldots + f_M(\boldsymbol{r})\psi_M(E)$$

Spatial distribution

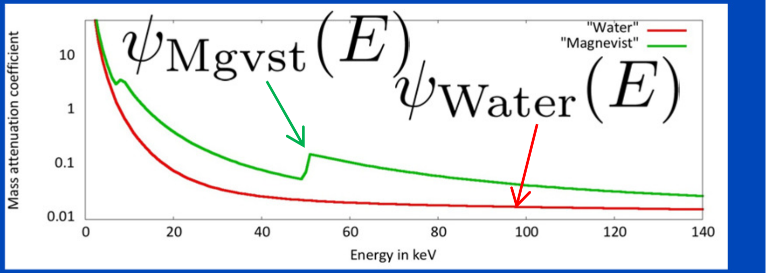
Spectral property

- **Example**

$f_{\text{Water}}(\boldsymbol{r})$



$f_{\text{Mgvst}}(\boldsymbol{r})$



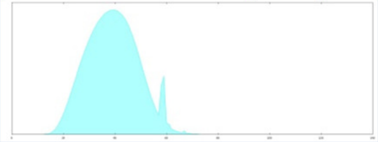
- **The x-ray transform**

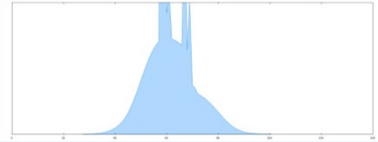
$$p_m = \mathrm{X} f_m(\boldsymbol{r})$$

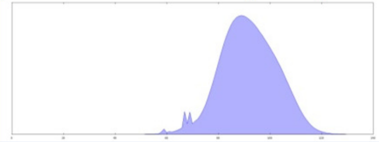
Sinogram domain

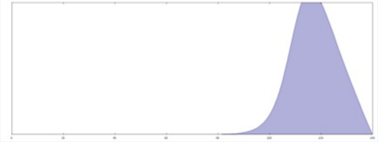
Image domain

Multiple Energy CT Example


$w_{\text{Bin1}}(E)$


$w_{\text{Bin2}}(E)$


$w_{\text{Bin3}}(E)$


$w_{\text{Bin4}}(E)$

$$q_1 = -\ln \int dE\, w_{\text{Bin1}}(E) e^{-p_{\text{Water}}\psi_{\text{Water}}(E) - p_{\text{Mgvst}}\psi_{\text{Mgvst}}(E)}$$

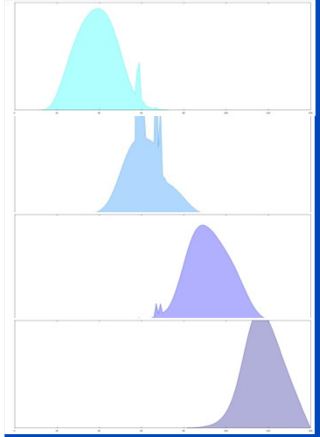
$$q_2 = -\ln \int dE\, w_{\text{Bin2}}(E) e^{-p_{\text{Water}}\psi_{\text{Water}}(E) - p_{\text{Mgvst}}\psi_{\text{Mgvst}}(E)}$$

$$q_3 = -\ln \int dE\, w_{\text{Bin3}}(E) e^{-p_{\text{Water}}\psi_{\text{Water}}(E) - p_{\text{Mgvst}}\psi_{\text{Mgvst}}(E)}$$

$$q_4 = -\ln \int dE\, w_{\text{Bin4}}(E) e^{-p_{\text{Water}}\psi_{\text{Water}}(E) - p_{\text{Mgvst}}\psi_{\text{Mgvst}}(E)}$$

Material-Selective Multiple Energy CT

Measuring the same object with B different detected spectra yields B different functions of the same material–selective sinograms. We aim to invert those measurements to get the material selective sino–grams, whose reconstructions are the material–selective images.



Measurement

$$q_1 = q_1(p_{\mathrm{Water}}, p_{\mathrm{Mgvst}})$$
$$q_2 = q_2(p_{\mathrm{Water}}, p_{\mathrm{Mgvst}})$$
$$q_3 = q_3(p_{\mathrm{Water}}, p_{\mathrm{Mgvst}})$$
$$q_4 = q_4(p_{\mathrm{Water}}, p_{\mathrm{Mgvst}})$$

Material–selective sinograms

$$p_{\mathrm{Water}} = p_1(q_1, q_2, q_3, q_4)$$
$$p_{\mathrm{Mgvst}} = p_2(q_1, q_2, q_3, q_4)$$

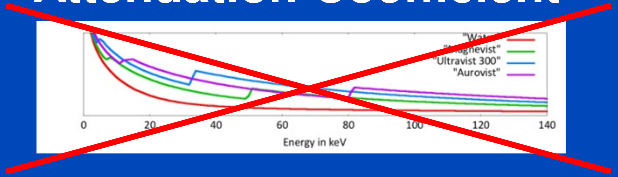
Reconstruction

$$f_{\mathrm{Water}} = \mathsf{X}^{-1} p_{\mathrm{Water}}$$
$$f_{\mathrm{Mgvst}} = \mathsf{X}^{-1} p_{\mathrm{Mgvst}}$$

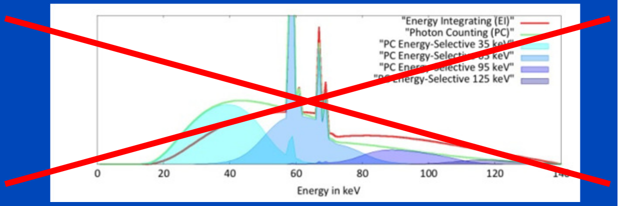
Empirical Methods

- **No knowledge required:**

Material's Mass Attenuation Coefficient



Detected Spectra



- **Instead: Direct calibration of the inversion formula**

Material–selective sinogram →

$$p_m(q_1, q_2, \ldots, q_B)$$

↖ **Measurements**

ECC	Empirical cupping correction	[MedPhys 33:1269, 2006]
ECCU	ECC with tube voltage modulation	[PMB 55:4107, 2010]
EDEC	Empirical dual energy calibration	[MedPhys 34:3630, 2007]
EMEC	Empirical multiple energy calibration	[Medical Imaging Conference MIC21.S-177, 2011], [RSNA, SSJ21-05, 2011]

EMEC Series Expansion

- Empirical multiple energy calibration (EMEC) uses the series expansion

$$p_m(q_1, q_2, ..., q_B) = \sum_{k_1,k_2,...,k_B} c_{m,k_1,k_2,...,k_B} q_1^{k_1} q_2^{k_2} ... q_B^{k_B}$$

  to obtain material–selective intersection lengths p_m from polychromatic measurements q_b.
- The unknowns $c_{m,k_1,k_2,...,k_B}$ are calculated from a calibration scan according.

Different Ways of EMEC

Number of detected spectra ↘

↙ Number of materials

- For $B>M$ redundant ways to calculate p_m exist.

$$p_{m\boxed{,w}}(q_1, q_2, \ldots, q_B) = \sum_{k_1,k_2,\ldots,k_B} c_{m\boxed{,w,}k_1,k_2,\ldots,k_B}\, q_1^{k_1} q_2^{k_2} \ldots q_B^{k_B}$$

- Binary notation
  - 1 source is used
  - 0 source is unused

  for an specific way.

➡

1100 means:

$$p_{\text{Water}} = p_1(q_1, q_2)$$

$$p_{\text{Mgvst}} = p_2(q_1, q_2)$$

$$f_{m,1100}(\boldsymbol{r}) = \sum_{k_1,k_2} c_{m,1100,k_1,k_2,0,0} \cdot f_{k_1,k_2,0,0}(\boldsymbol{r})$$

Calibration using the Yin Yang Phantom

w	Bin 1	Bin 2	Bin 3	Bin 4	D^2_{Water}	$D^2_{\text{Magnevist}}$
1100	X	X	0	0	14.3	4.4
1010	X	0	X	0	12478	6109
0110	0	X	X	0	2.6	1.8
1001	X	0	0	X	3.7	2.1
0101	0	X	0	X	1.5	1.3
0011	0	0	X	X	0.9	0.9

Way 1010 is expected to fail in doing the material separation. EMEC automatically finds that the spectral spearation is too low using that way.

$$f_{m,w}(\boldsymbol{r}) = \sum_{k_1,k_2,\ldots,k_B} c_{m,w,k_1,k_2,\ldots,k_B} f_{k_1,k_2,\ldots,k_B}(\boldsymbol{r})$$

$$D^2_{m,w} = \int d^3\boldsymbol{r} g(\boldsymbol{r})(f_{m,w}(\boldsymbol{r}) - t_m(\boldsymbol{r}))^2$$

Dose Minimization (I)

- Combine all redundant ways w to one material–selective rawdata set:

Linear combination

$$p_m = \sum_w h_{m,w} p_{m,w}(q_1, q_2, \dots, q_B)$$

Normalization

$$1 = \sum_w h_{m,w}$$

Noise model

$$\mathrm{Var}\ q_b \longrightarrow \mathrm{Var}\ p_m$$

We use a noise model for each measured sinogram bin q_b. Hence, error propagation can be used to calculate the variance of p_m. Recall that $p_{m,w}$ is just a series expansion and thus error propagation is an easy task.

Dose Minimization (II)

Solving
$$\min_{h_{m,w}} \mathrm{Var}\, p_m$$
yields
$$h_{m,w}$$
and thus
$$p_m = \sum_w h_{m,w} p_{m,w}$$
minimizes the pixel noise in the material–selective sinogram.

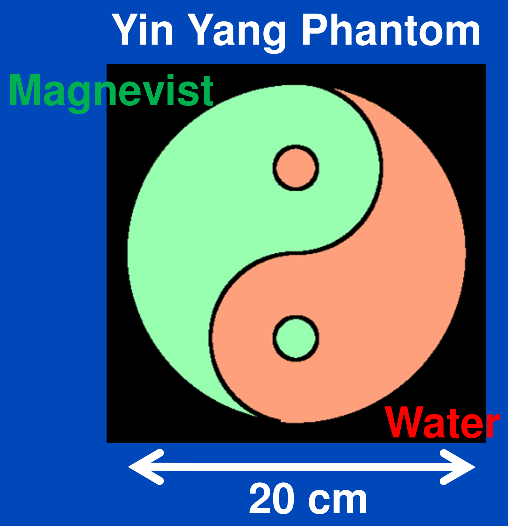
We minimize the variance of each material–selective sinogram pixel p_m separately on-the-fly. Since all $p_{m,w}$ have the same value up to the noise, we may use completely different $h_{m,w}$ values for each sinogram entry.

- Reconstruction yields the material–selective images:

$$f_{\mathrm{Water}} = \mathsf{X}^{-1} p_{\mathrm{Water}}$$
$$f_{\mathrm{Mgvst}} = \mathsf{X}^{-1} p_{\mathrm{Mgvst}}$$

Simulations

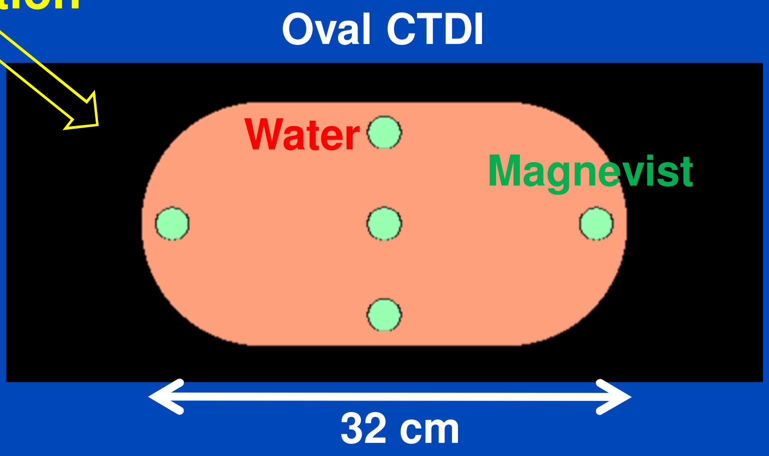
Calibration



- Analytical 2D simulation
  - fan–beam geometry
  - 512 projections, 512 rays per projection
  - Poisson noise model $\mathrm{Var}\ q_b$

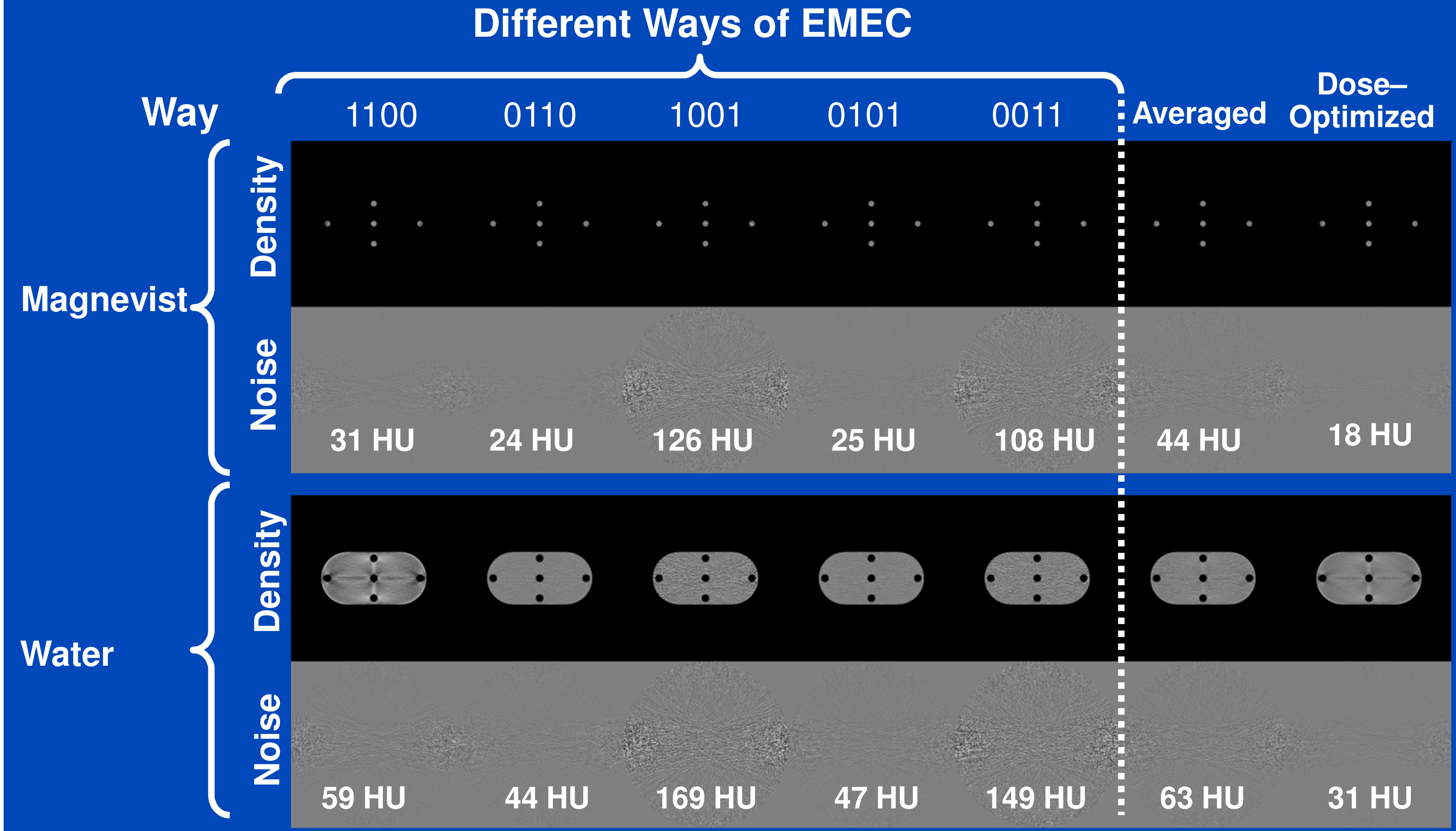
- Tucker spectrum
  - 140 kV
  - 2 mm Al prefiltration
  - 1.6 mm CdZnTe detectors

Application



Way 1010 was found to be inappropriate by the EMEC calibration (above) and is therefore excluded in the following consideration.

Application Phantom Results



Numbers show the variance in the difference image of two independent noise realizations

C/W = 0/50 HU

Summary

- Energy–selective CT systems offer redundant ways to reconstruct material–selective CT images.
  - Here we used EMEC* (empirical multiple energy calibration) to calibrate each way.

- Dose Minimization
  - Combines redundant ways for minimal noise
  - Patient specific, sinogram-pixel specific
  - Reduces image noise by ~25% with respect to the best single way

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*N. Maass, S. Sawall, M. Kachelriess, “Empirical Multiple Energy Calibration for Material–Selective CT,” MIC21.S-177, MIC 2011.

Thank You!

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