Dose Minimization for Material–Selective CT with Energy–Selective Detectors

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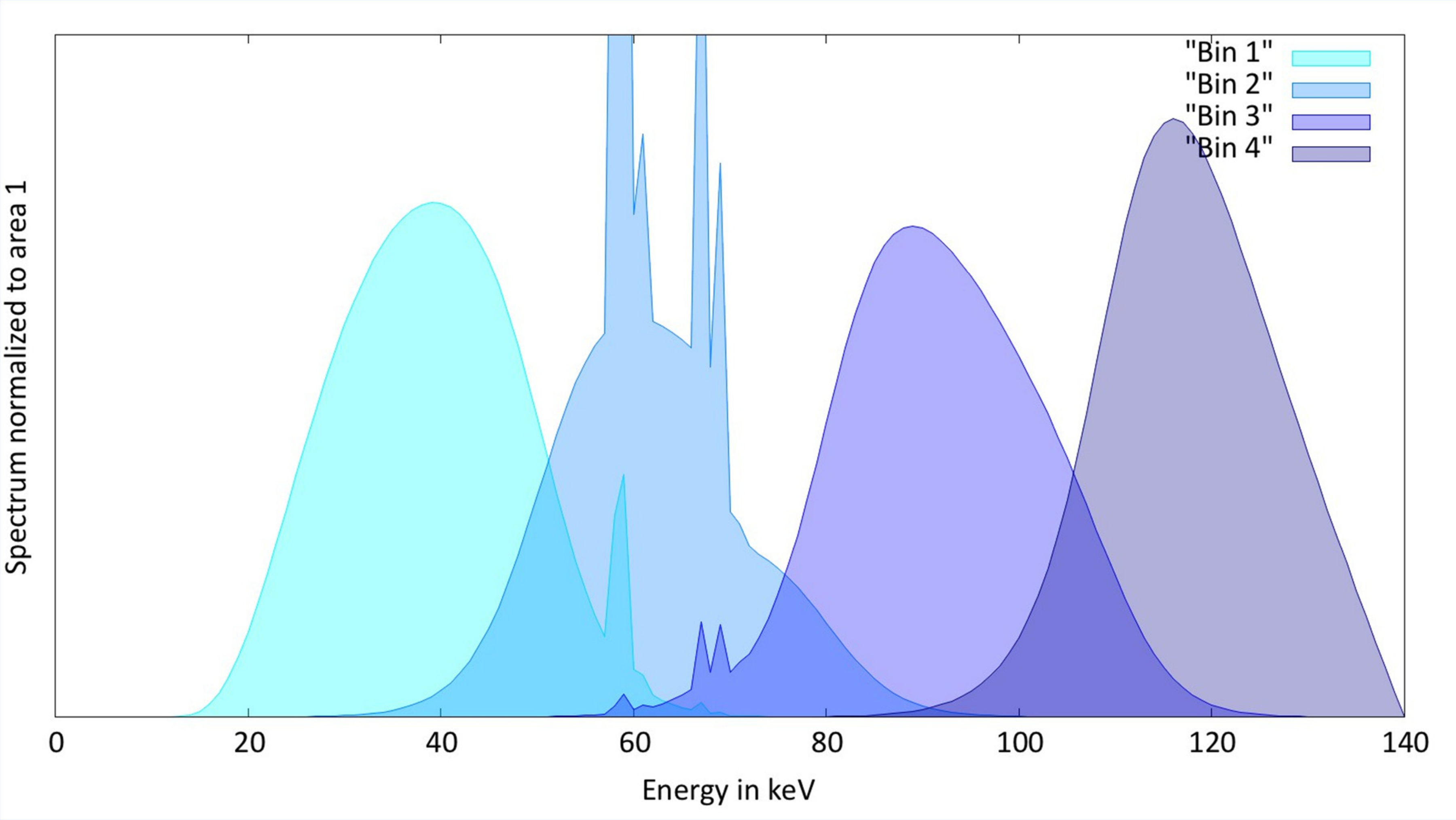
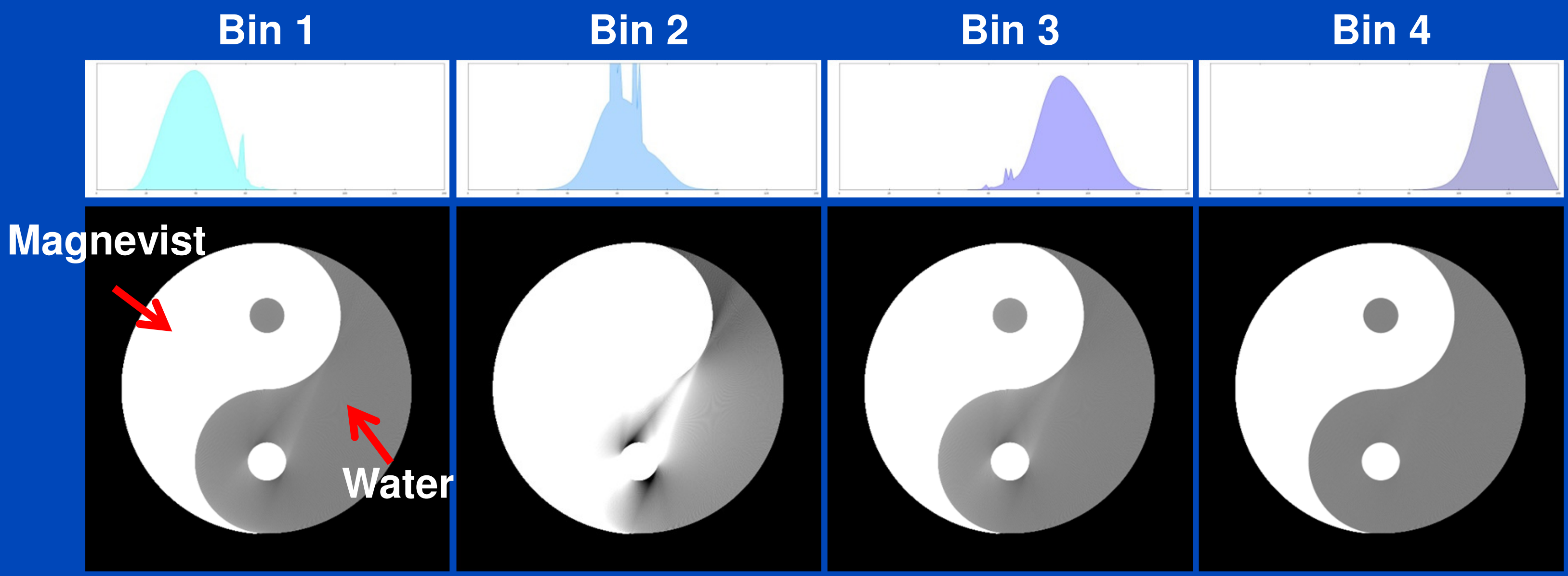
Energy–Selective CT



Energy–Selective CT Images



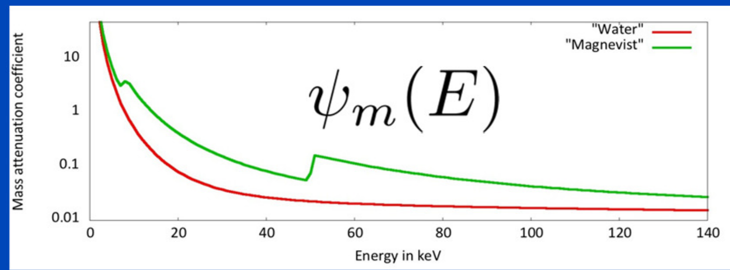
C/W = 0/100 HU

Multiple Energy CT

- **The object consists of M independent materials:**

$$\mu(\boldsymbol{r}, E) = f_1(\boldsymbol{r})\psi_1(E) + f_2(\boldsymbol{r})\psi_2(E) + ... + f_M(\boldsymbol{r})\psi_M(E)$$

$f_{\mathrm{Water}}(\boldsymbol{r})$   $f_{\mathrm{Mgvst}}(\boldsymbol{r})$



- **The x-ray transform**

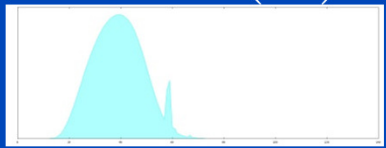
$$p_m = \mathsf{X} f_m(\boldsymbol{r})$$

- **Measurement with B different detected spectra**

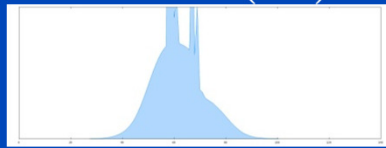
$$q_b = -\ln \int dE w_b(E) e^{-p_1\psi_1(E) - p_2\psi_2(E) - \ldots - p_M\psi_M(E)}$$

Multiple Energy CT Example

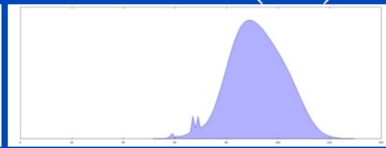
$w_{\mathrm{Bin1}}(E)$



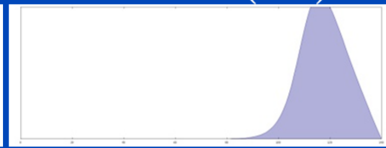
$w_{\mathrm{Bin2}}(E)$



$w_{\mathrm{Bin3}}(E)$



$w_{\mathrm{Bin4}}(E)$



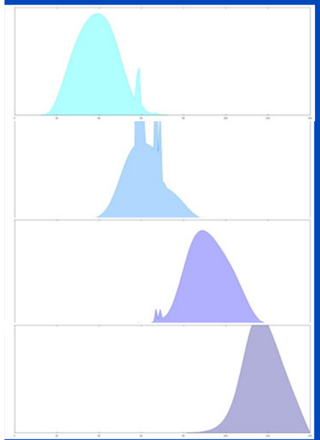
$$q_1 = -\ln \int dE\, w_{\mathrm{Bin1}}(E) e^{-p_{\mathrm{Water}}\psi_{\mathrm{Water}}(E) - p_{\mathrm{Mgvst}}\psi_{\mathrm{Mgvst}}(E)}$$

$$q_2 = -\ln \int dE\, w_{\mathrm{Bin2}}(E) e^{-p_{\mathrm{Water}}\psi_{\mathrm{Water}}(E) - p_{\mathrm{Mgvst}}\psi_{\mathrm{Mgvst}}(E)}$$

$$q_3 = -\ln \int dE\, w_{\mathrm{Bin3}}(E) e^{-p_{\mathrm{Water}}\psi_{\mathrm{Water}}(E) - p_{\mathrm{Mgvst}}\psi_{\mathrm{Mgvst}}(E)}$$

$$q_4 = -\ln \int dE\, w_{\mathrm{Bin4}}(E) e^{-p_{\mathrm{Water}}\psi_{\mathrm{Water}}(E) - p_{\mathrm{Mgvst}}\psi_{\mathrm{Mgvst}}(E)}$$

Material–Selective Multiple Energy CT



Measurement

$$q_1 = q_1(p_{\text{Water}}, p_{\text{Mgvst}})$$
$$q_2 = q_2(p_{\text{Water}}, p_{\text{Mgvst}})$$
$$q_3 = q_3(p_{\text{Water}}, p_{\text{Mgvst}})$$
$$q_4 = q_4(p_{\text{Water}}, p_{\text{Mgvst}})$$

Material–selective sinograms

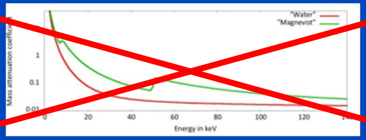
$$p_{\text{Water}} = p_1(q_1, q_2, q_3, q_4)$$
$$p_{\text{Mgvst}} = p_2(q_1, q_2, q_3, q_4)$$

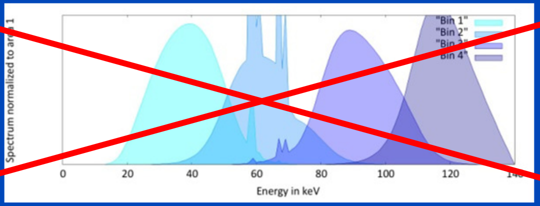
Reconstruction

$$f_{\text{Water}} = \mathsf{X}^{-1} p_{\text{Water}}$$
$$f_{\text{Mgvst}} = \mathsf{X}^{-1} p_{\text{Mgvst}}$$

Empirical Calibration

- No knowledge required: Material’s mass attenuation coefficient & detected spectra





- Instead: Direct calibration of the inversion formula

$$p_m(q_1, q_2, ..., q_B)$$

<u>**Poster MIC21.S-177**</u>

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Empirical Multiple Energy Calibration for Material–Selective CT

EMEC Series Expansion

- Empirical multiple energy calibration (EMEC) uses the series expansion

$$p_m(q_1, q_2, \ldots, q_B) = \sum_{k_1, k_2, \ldots, k_B} c_{m,k_1,k_2,\ldots,k_B} q_1^{k_1} q_2^{k_2} \ldots q_B^{k_B}$$

Material–selective sinograms

$$p_{\mathrm{Water}} = p_1(q_1, q_2, q_3, q_4)$$
$$p_{\mathrm{Mgvst}} = p_2(q_1, q_2, q_3, q_4)$$

are calculated from a

Different Ways of EMEC

- For $B>M$ redundant ways to calculate p_m exist.

$$p_{m,w}(q_1, q_2, ..., q_B) = \sum_{k_1,k_2,...,k_B} c_{m,w,k_1,k_2,...,k_B} q_1^{k_1} q_2^{k_2} \dots q_B^{k_B}$$

- Binary notation
  - 1 means "bin is used"
  - 0 means "bin is not used"

  for an specific way.

1100 means:

$$p_{\text{Water}} = p_1(q_1, q_2)$$

$$p_{\text{Mgvst}} = p_2(q_1, q_2)$$

$$f_{m,1100}(\boldsymbol{r}) = \sum_{k_1,k_2} c_{m,1100,k_1,k_2,0,0} \cdot f_{k_1,k_2,0,0}(\boldsymbol{r})$$

FAU dkfz.

Dose Minimization (I)

- **Combine all redundant ways w to one material–selective rawdata set:**

Linear combination $$p_m = \sum_w h_{m,w} p_{m,w}(q_1, q_2, \ldots, q_B)$$

Normalization $$1 = \sum_w h_{m,w}$$

Noise model $$\mathrm{Var}\ q_b \longrightarrow \mathrm{Var}\ p_m$$

Dose Minimization (II)

Solving

$$\min_{h_{m,w}} \mathrm{Var}\, p_m$$

yields

$$h_{m,w}$$

and thus

$$p_m = \sum_w h_{m,w} p_{m,w}$$

minimizes the pixel noise in the material–selective sinogram. This is done sinogram pixel-specific and patient-specific!

- **Reconstruction yields the material–selective images:**

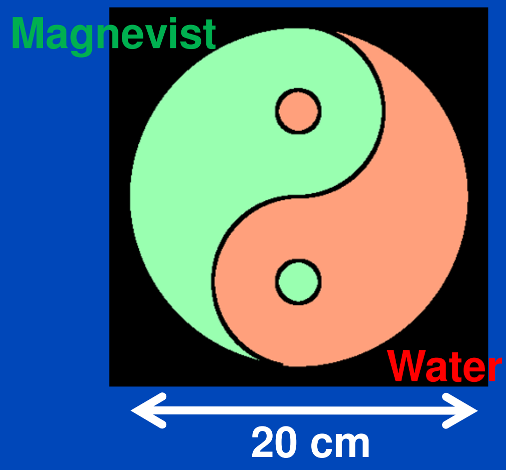
$$f_{\mathrm{Water}} = \mathsf{X}^{-1} p_{\mathrm{Water}}$$

$$f_{\mathrm{Mgvst}} = \mathsf{X}^{-1} p_{\mathrm{Mgvst}}$$

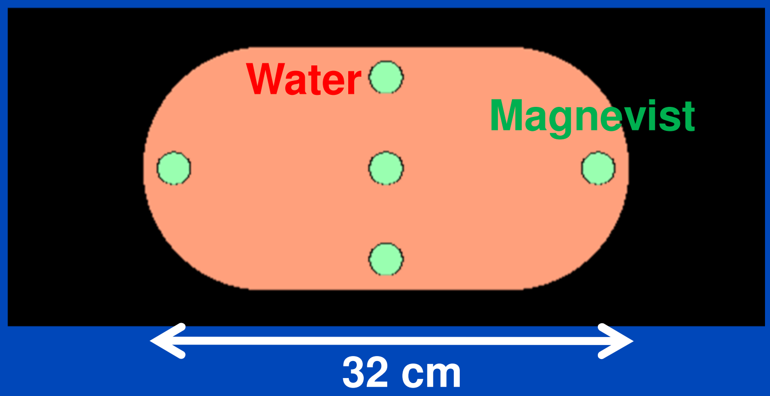
Simulations

- **Analytical 2D simulation**
  - fan–beam geometry
  - 512 projections, 512 rays per projection
  - Poisson noise model $\text{Var}\ q_b$
- **Tucker spectrum**
  - 140 kV
  - 2 mm Al prefiltration
  - 1.6 mm CdZnTe detectors

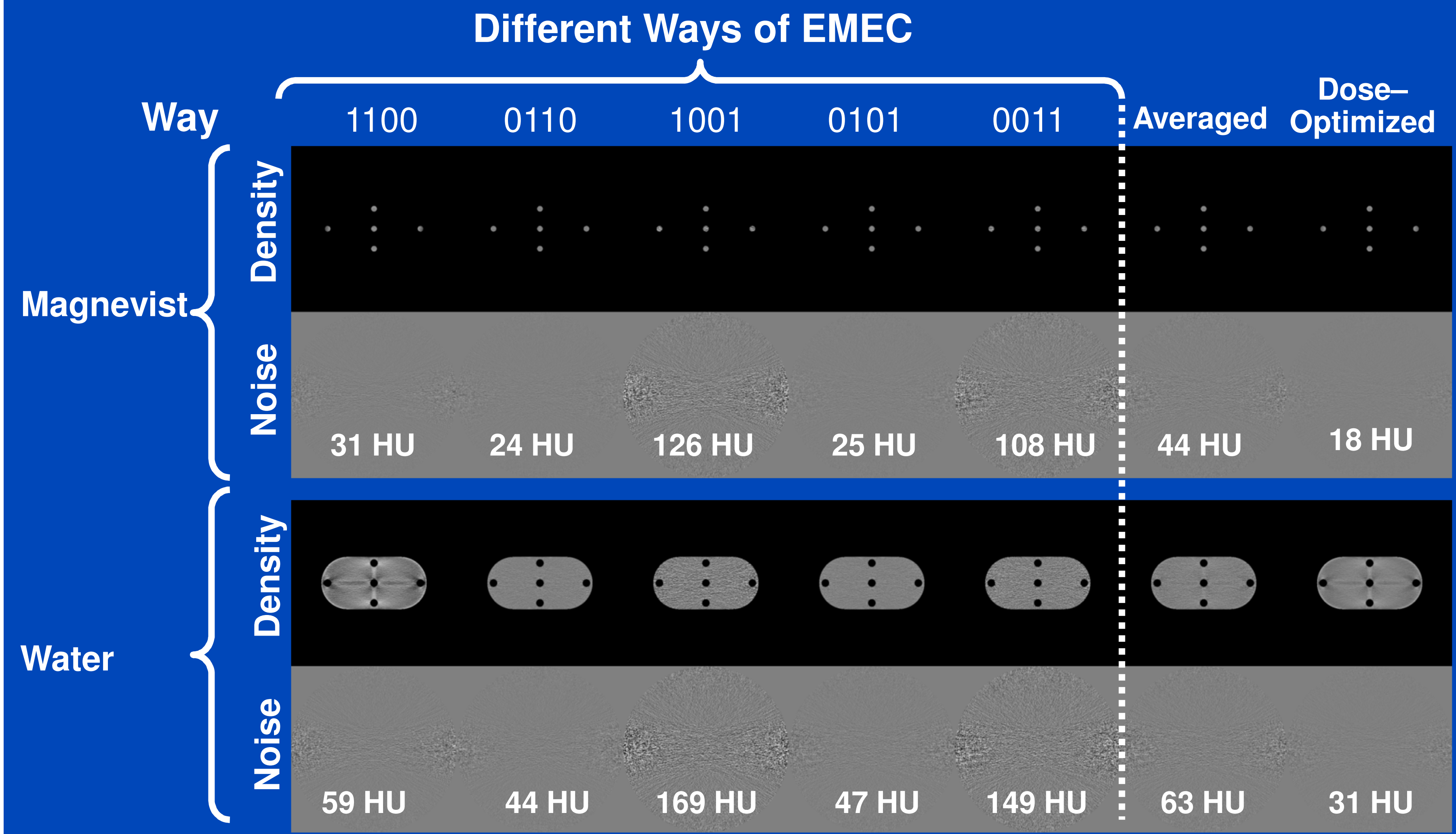
Yin Yang Phantom



Oval CTDI



CTDI Phantom Results



Numbers show the variance in the difference image of two independent noise realizations

C/W = 0/50 HU

Summary

- **Energy–selective CT systems offer redundant ways to reconstruct material–selective CT images.**
  - **Here we used EMEC[*] (empirical multiple energy calibration) to calibrate each way.**

- **Dose Minimization**
  - **Combines redundant ways for minimal noise**
  - **Patient specific, sinogram-pixel specific**
  - **Reduces image noise by ~25% with respect to the best single way**

[*]N. Maass, S. Sawall, M. Kachelriess, “Empirical Multiple Energy Calibration for Material–Selective CT,” MIC21.S-177, MIC 2011.

Thank You!

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