Attenuation-Based Reconstruction of Low and High Frequency Components of Simulated Detected X-Ray Spectra

C. Leinweber[1,2], J. Maier[1,2], S. Sawall[1], H. Stoschus[2], F. Ballach[2], T. Müller[2], M. Hammer[2], R. Christoph[2], and M. Kachelrieß[1]

[1]Medical Physics in Radiology, German Cancer Research Center (DKFZ), Heidelberg, Germany
[2]Werth Messtechnik GmbH, Gießen, Germany

Correspondence to: carsten.leinweber@dkfz.de

Introduction

Detailed knowledge about the x-ray detected spectrum is of importance for many CT applications and artifact correction methods (e.g. dose estimation, beam hardening correction, scatter correction, dual energy decomposition). An attractive strategy to access the spectrum is by reconstruction from transmission measurements. This inverse problem has proven to be highly ill-conditioned even under ideal assumptions. Currently, there exist several methods that provide a solution to the reconstruction problem, e.g. truncated singular value decomposition (TSVD) [1], expectation-maximization (EM) [2], neural networks and few-parameter modelling. They appear to either result in unphysical spectra or to strongly depend on initial start values and stopping criteria. In this work we describe a new approach to reconstruct the detected spectrum by combining a TSVD solution with minimal prior knowledge about the x-ray generating material, providing a spectrum that accurately reproduces the incident transmission measurements while exhibiting a physically reliable shape. We call this method prior truncated singular value decomposition (PTSVD).

Materials and Methods

Results from TSVD show that unphysical oscillations are introduced into the reconstructed spectrum when the incident spectrum exhibits high spectral frequencies like characteristic peaks or k-edges. These oscillations can be attributed to the low dimensionality of the singular vectors that are involved in the TSVD solution.

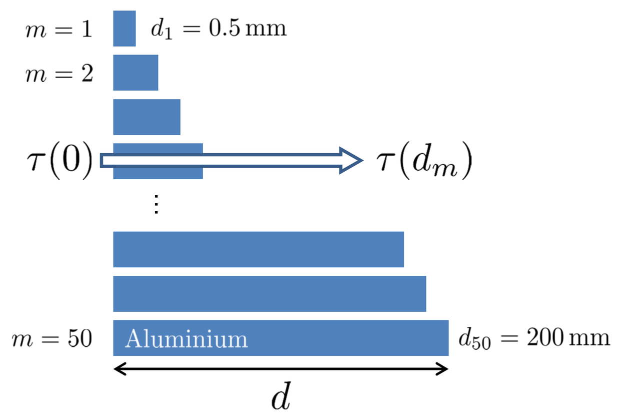
The new approach of PTSVD is to utilize the higher dimensionality of the remaining singular vectors from null space to calculate a solution that involves high frequencies while providing a shape that better represents the physical nature of bremsstrahlung-induced radiation. We start with an initial guess of the high frequency components as a prior spectrum and calculate the linear combination of the singular vectors from null space that represents this prior best. As the singular vectors span an orthonormal space, this best approximation corresponds to the projection of the prior spectrum onto the null space. In the next step we assume that the deviations between the true spectrum and the TSVD solution can be attributed to this representation of the high frequency components. If the prior spectrum involves the true high frequency contributions the sum of the TSVD solution and the null space solution thus equals the true spectrum.

In order to obtain the true prior spectrum we iteratively minimize a cost function that accounts for the physical assumption on the reconstructed spectrum. These are non-negativity and flatness of the low frequency components. A spectrum that represents the low frequency components is obtained by subtracting the prior spectrum containing the high frequency components from the PTSVD solution.

Motivation

Transmission measurements

$$\tau(d) = \frac{N}{N_0} = \int_E dE\, w(E)\, e^{-\mu(E)\, d}$$



Discretization

$$\tau_m = \sum_{b=1}^{B} w_b\, e^{-\mu_{mb}\, d_m}$$

System matrix

$$A_{mb} = e^{-\mu_{mb}\, d_m}$$

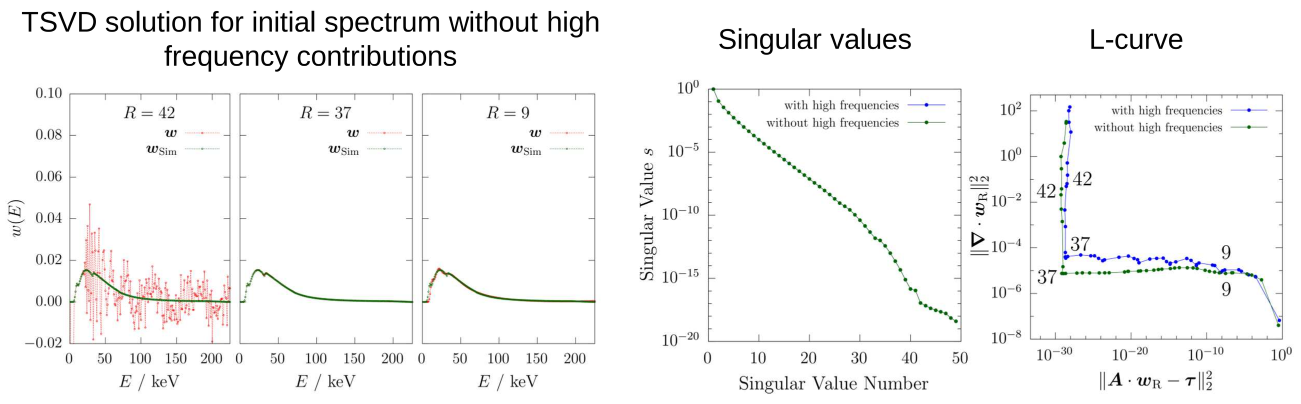
Problem of x-ray spectrum reconstruction

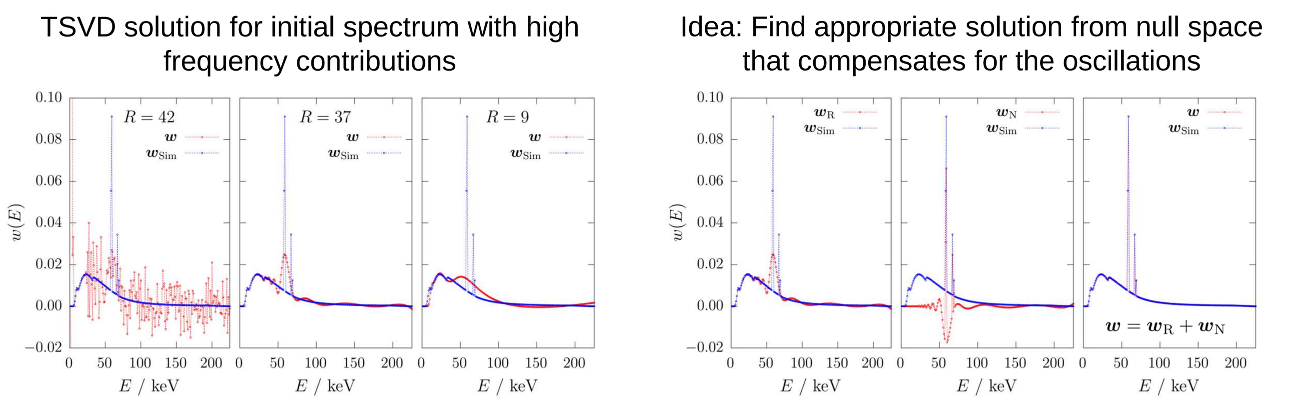
$$\boldsymbol{w} = \boldsymbol{A}^{-1}\boldsymbol{\tau}.$$

Truncated singular value decomposition (TSVD)

$$\boldsymbol{A} = \sum_{b=1}^{B} \boldsymbol{u}_b \cdot s_b \boldsymbol{v}_b^T \longrightarrow \boldsymbol{w}_\mathrm{R} = \left(\sum_{b=1}^{R} \boldsymbol{v}_b \cdot \frac{\boldsymbol{u}_b^T}{s_b}\right)\cdot\boldsymbol{\tau} \quad \text{with} \quad R \le B$$

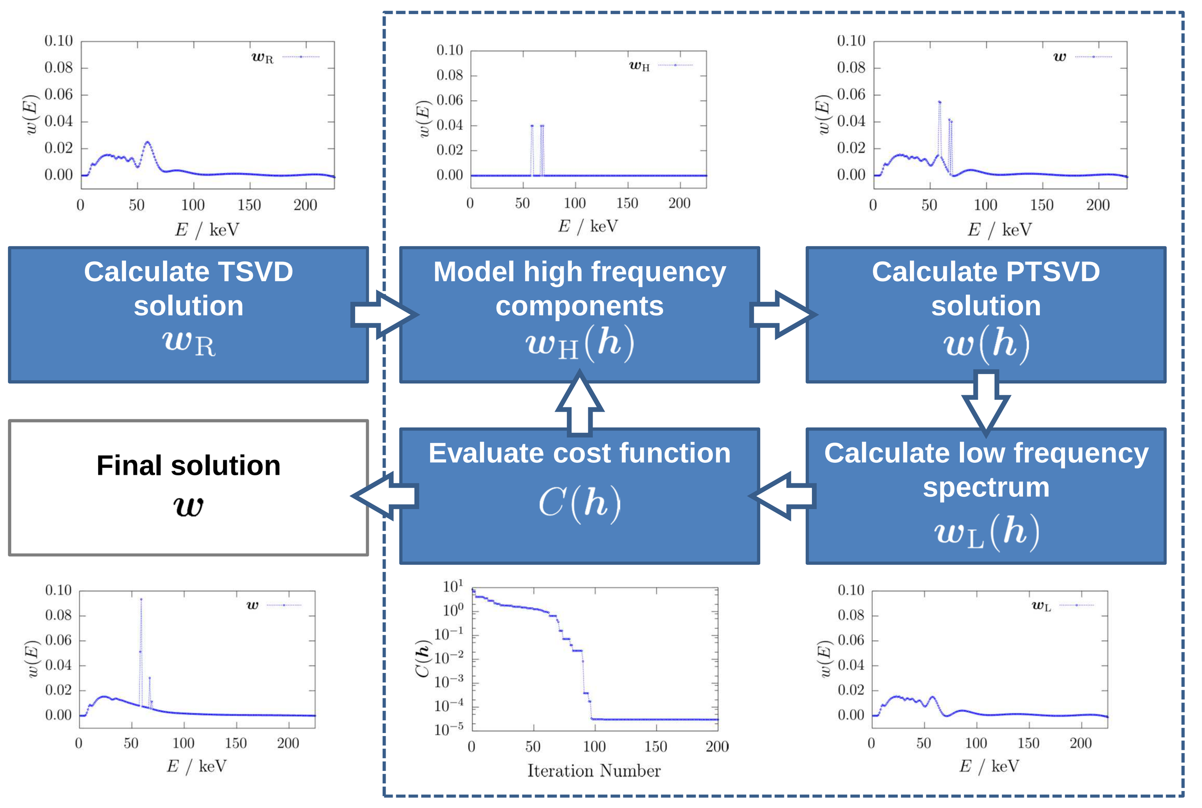
Numeric analysis





Workflow

Prior truncated singular value decomposition (PTSVD)



Spectrum reconstruction

$$\boldsymbol{w}(\boldsymbol{h}) = \underbrace{\sum_{b=1}^{R}\left(\frac{\boldsymbol{u}_b^T\cdot\boldsymbol{\tau}}{s_b}\right)\boldsymbol{v}_b}_{\substack{\boldsymbol{w}_\mathrm{R} \\ \text{Solution from range (TSVD)}}} + \underbrace{\sum_{b=R+1}^{B}\left(\boldsymbol{v}_b^T\cdot\boldsymbol{w}_\mathrm{H}(\boldsymbol{h})\right)\boldsymbol{v}_b}_{\substack{\boldsymbol{w}_\mathrm{N} \\ \text{Solution from null space}}}$$

Model of high frequency components

$$\boldsymbol{w}_\mathrm{H}(\boldsymbol{h}) = \sum_{p=1}^{P} h_p\, \boldsymbol{e}_p$$

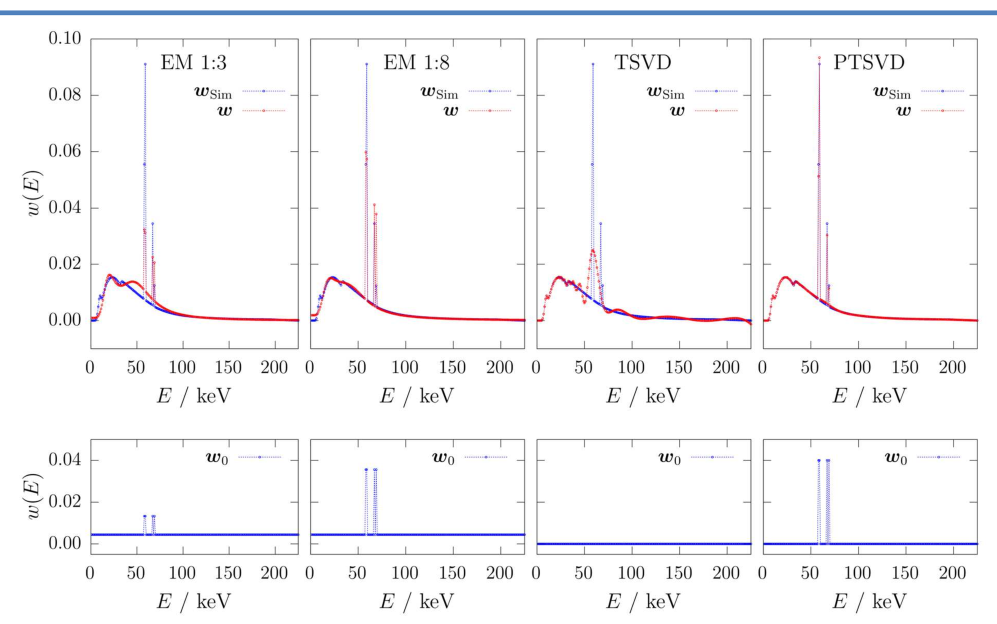
Cost function

$$C(\boldsymbol{h}) = \|\boldsymbol{w}(\boldsymbol{h}) \wedge \boldsymbol{0}\|_2^2 + \lambda\,\|\boldsymbol{\nabla}\cdot\boldsymbol{w}_\mathrm{L}(\boldsymbol{h}))\|_2^2 \quad \text{with} \quad \boldsymbol{w}_\mathrm{L}(\boldsymbol{h}) = \boldsymbol{w}(\boldsymbol{h}) - \boldsymbol{w}_\mathrm{H}(\boldsymbol{h})$$

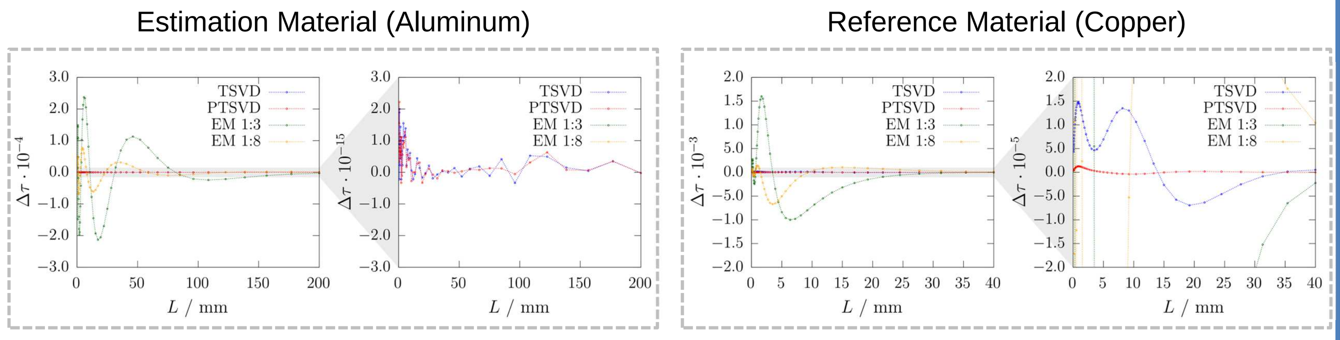
Results

Reconstructed spectra

Initial prior spectrum



Transmission error $\Delta\tau = \tau_\mathrm{Sim} - \tau_\mathrm{Est}$



solution.

In this work we test the new method for simulated data. A detected x-ray spectrum is simulated according to Tucker et al. assuming a maximum acceleration voltage of 225 kV and a tungsten target with characteristic peaks at 58.0 keV, 59.3 keV, 67.2 keV and 69.1 keV. The detected spectrum further accounts for the detector response of a CsI:Tl scintillator of 0.6 mm thickness. We include 50 transmission measurements into the reconstruction process with aluminum as absorber and attenuation lengths ranging from 0.5 mm to 200 mm. In order to test the reconstructed spectra we simulate 50 additional transmission measurements with copper ranging from 0.1 mm to 40 mm. These reference measurements are not included into the reconstruction process. Attenuation coefficients are taken from Cullen et al.

We compare PTSVD to the widely used EM approach of Sidky et al. The initial spectrum for the first EM iteration is chosen such that the amount of prior knowledge is similar to that incorporated into the PTSVD method. We show two scenarios of the EM algorithm which differ in the background-to-peak ratio of the initial spectrum. 5000 EM iterations where performed in both scenarios providing the best results.

Results

The spectrum reconstructed with PTSVD is in good accordance with the true simulated spectrum. The method comparison reveals that PTSVD outperforms the EM method in case of ideal input data. The differences between the results of the two EM scenarios show a strong dependence on the initial value. We found that the EM method diverges when increasing the number of iterations, resulting in unphysical spectra. In comparison PTSVD shows good convergence properties. Transmission values simulated with the solution from TSVD intrinsically show the smallest deviations to the true transmission values for the material incorporated into the reconstruction process. A comparison shows that PTSVD conserves this property. For the reference material the results point out that the transmission values simulated with help of the PTSVD solution show deviations to the true transmission values which are one order of magnitude smaller than those simulated with TSVD. In comparison to EM the PTSVD method leads to a decrease in the maximum deviation to the true transmission values in the order of two magnitudes. Overall the comparison reveals that an accurate reconstruction is essential for the simulation of materials that have not been incorporated into the reconstruction process.

Acknowledgements

This study was supported by the Arbeitsgemeinschaft industrieller Forschungsvereinigungen (AiF) under Grant No. KF2301007NT3. Parts of the x-ray simulation software were provided by RayConStruct® GmbH, Nürnberg, Germany.

[1] S. Tominaga, "A Singular Value Decomposition Approach to X-Ray Spectral Estimation from Attenuation Data", vol. 243, pp. 530–538, 1986.

[2] E. Y. Sidky, L. Yu, X. Pan, Y. Zou, and M. Vannier, "A robust method of x-ray source spectrum estimation from transmission measurements: Demonstrated on computer simulated, scatter-free transmission data", Journal of applied physics, vol. 97, no. 12, p. 124701, 2005.